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CONSIDERATIONS ON RIGID BODY MECHANICS

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Abstract:
This article examines the theoretical foundations and understanding of solid mechanics that should be taught in depth in engineering schools, and provides motivation for continued independent study of this topic using the example of a gyroscope.

Key words and phrases: torque, moment of inertia, angular momentum, law of conservation of momentum, gyroscope, gyroscopic effect, procession, gyrocompass, gyrohorizon, gyrovertikl, gyrostabilizer.

Аннотация:
В данной статье рассматриваются теоретические основы и понимание механики твердого тела, которые должны углубленно преподаваться в инженерных вузах, а также дается мотивация к продолжению самостоятельного изучения этой темы на примере гироскопа.

Ключевые слова и фразы: вращательный момент, момент инерции, момент импульса, закон сохранения импульса, гироскоп, гироскопический эффект, процессия, гирокомпас, гирогоризонт, гировертикал, гиростабилизатор.

As is well known, it is advisable for military universities to study military-technical and combat vehicles and mechanisms, combat weapons, their operating principles and the theoretical principles on which they are based, as well as theoretical solutions for their improvement and their verification by practical results. To perform such work, it would be useful for a scientific researcher to have a good knowledge of solid mechanics as part of a general physics course. This is because machine parts and mechanisms made of solids primarily undergo rotational motion.

It will be possible to write down the dynamic and kinematic laws of motion of rigid bodies, that is, the differential and kinematic equations of their motion, and draw conclusions based on them. To do this, it is necessary to know the fundamental equation of rotational motion dynamics[1]:

$$\vec{M} = \frac{d\vec{L}}{dt} \quad \text{va} \quad \vec{M} = I \cdot \vec{\varepsilon} \qquad (1)$$

In these equations $\vec{M} -$ torque or torque moment, $\vec{L} -$ angular momentum, I-moment of inertia, $\vec{\varepsilon} -$ angular acceleration.

To write the equation of the dynamics of rotational motion or the differential equation of rotational motion, we find the sum of the moments of forces acting on the solid part of the body under study, relative to the axis around which this solid part of the body rotates, equating it to $I \cdot \vec{\varepsilon}$. From this it is clear that, consequently, the moment of force in the original equation (1) must be represented as a sum of moments, that is, as follows:

$$\vec{M} = \sum_{i-1}^{n} \vec{M_\iota} \qquad (2)$$

It is clear that the resulting differential equation is a second-order differential equation, since we know that the angular acceleration is the second-order derivative of the variable rotation angle. In addition, this equation may also contain terms with first-order derivatives; for example, such a term may arise from the friction force, since the friction force depends on the speed, and the speed is the first-order derivative of this angle. By solving the resulting second-order differential equation and integrating it twice, we obtain the kinematic equation of motion. The resulting kinematic equation is an equation for the dependence of this angle on time, that is, in the form $\varphi = f(t)$. From this equation, the form of the motion is clearly visible. Therefore, based on this equation, theoretical conclusions can be drawn about the occurring or observed motion.

Above, we considered two-dimensional motion, that is, the motion of a part of a rigid body with a degree of freedom equal to 1. By the general motion of a rigid body, we mean the motion of a rigid body at a given moment, consisting of the sum of the translational and rotational motions of its center of gravity around its center of gravity. In this case, the number of degrees of freedom of a rigid body is 6, which corresponds to three linear coordinates x, y, z, representing translational motion, and three angular coordinates φ, θ, ψ, representing rotational motion. It is clear that there are also 3 differential and kinematic equations, that is, 6 in total. In special cases, the number of equations can be reduced: for example, when there is only translational motion, there can be a maximum of 3, 2, or even just one. If the rigid body is stationary and moves only in a circle, the number of equations may be three, two, or even just one. Even in such complex motions, differential equations are first written, and then, by solving them, kinematic equations and even the equation of the trajectory of motion are found. Only then can theoretical conclusions about motion be drawn from these kinematic and trajectory equations. In this case, the fundamental equation of translational motion dynamics is used for translational motion[2]:

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{va} \quad \vec{F} = m\vec{a} \qquad (3)$$

The main component of a rigid body's motion is rotational motion. Therefore, we will primarily consider the rotational motion of a rigid body. Let's begin with the quantities and laws of motion dynamics:

$\vec{M} = I \cdot \vec{\varepsilon}$ - the fundamental equation of rotational dynamics,

$\vec{L} = I \cdot \vec{\omega}$ - angular momentum expression

$E = \frac{I\omega^2}{2}$ - expression for the kinetic energy of rotational motion.

It's clear from these that the moment of inertia plays a role in each of them. Therefore, when we consider the laws of rotational motion, we must first understand the moment of inertia. To this end, if we recall the manifestations of these expressions in translational motion, we will believe that in each of them, mass is used instead of the moment of inertia. If we consider the role of mass in translational motion, we will recall that it is a measure of inertia. It follows that in rotational motion, the moment of inertia is also a measure of inertia. This means that the greater the mass of bodies moving in translation, the more difficult it is to set them in motion and, if they are moving, to stop them. Similarly, the greater the moment of inertia of bodies moving in rotational motion, the more difficult it is to set them in motion and, if they are moving, to stop them[3].

Determining body weight is straightforward: it can be determined by weighing the body on a scale. Weight can only change if a body part is removed, a new part is added, or if a significant amount of time has passed since the measurement. As for the moment of inertia, which is a measure of the inertia of rotational motion, if the body has a regular geometric shape, it can be expressed by the following expressions:

$$I = \sum_{i=1}^{n} \Delta m_i \cdot r_i^2 \quad \text{va} \quad I = \int_0^m r^2 \cdot dm \qquad (4)$$

It can be found analytically, that is, by calculation, or from reference books. If the shape of the part of the solid being studied is not a regular geometric figure, it is found experimentally.

Regarding conservation laws in mechanics, there are laws of conservation of energy, momentum, and angular momentum. Of these, the laws of conservation of energy and angular momentum are characteristic of rotational motion. Of these, the law of conservation of energy is not particularly difficult to apply. The law of conservation of angular momentum is much more difficult to apply and understand. For angular momentum to be conserved, according to equation (1), the sum of the moments of the forces acting on the rigid body whose motion is being studied must be equal to zero. Only then, from a mathematical point of view, is the law of conservation of angular momentum fulfilled, that is, in order for the derivative of the angular momentum with respect to time to be equal to zero, the angular momentum must be unchanged, that is, constant:

$$\vec{M} = \sum_{i-1}^{n} \vec{M_\imath} = 0 \quad \text{da} \quad \vec{L} = \sum_{i-1}^{n} \vec{L_\imath} = \text{const} \qquad (5)$$

We know that the expression L ⃗=I·ω ⃗ is written for one body and if we conduct an experiment on one body, then, since ∑_(i=1)^n▒(L_i) ⃗ =const., it follows that with an increase in the moment of inertia, the angular velocity decreases, and with a decrease in the moment of inertia, the angular velocity increases. If a person is standing on a stationary platform (support platform) and the person on it begins to walk perpendicular to the diameter of the platform, then the platform will also move in the direction opposite to his movement, that is, the law of conservation of angular momentum is fulfilled.

If we consider a gyroscope as an example of the application of the law of conservation of angular momentum, then the law of conservation of angular momentum creates a gyroscopic effect. To explain the physics of this phenomenon, let's return to the expression of the law of conservation of angular momentum[4]:

$\vec{L} = I \cdot \vec{\omega}$=const

Accordingly, since angular momentum is a vector quantity, both its magnitude and direction must be conserved. As can be seen from the expression for the conservation law, since the moment of inertia is a scalar quantity, the direction of angular momentum coincides with the direction of angular velocity. The direction of the angular velocity coincides with the direction of the axis of the gyroscope's circular housing. In Figure 1, the gyroscope housing is shown in yellow and is called the rotor, and its axis of rotation is vertical. This means that when the gyroscope housing rotates horizontally, the axis of rotation remains vertical.

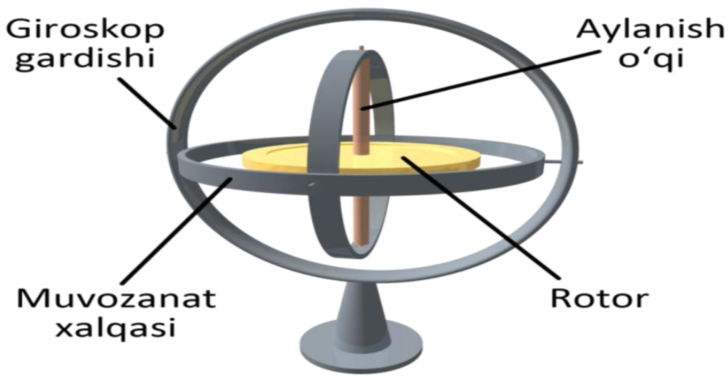

Fig. 1. Gyroscope view

Gyroscopes are used in various gyroscopic navigation systems (gyrocompass, gyrostabilizer, aircraft gyrohorizon, gyrovertical, ship roll stabilization, automatic ship control, as an automatic missile control device, as a course indicator, ensuring the stability of the projectile flight, For example, a gyroscope-based device in a tank is used for automatic control of a gun mounted on it, stabilization of the position of the tank's machine gun while moving, automatic movement control, stabilization of weapons, and determination of the location of a moving object. It should also be noted that gyroscopic devices are a key element in the guidance systems of cutting-edge, high-precision weapons. For example, to improve the firing efficiency of anti-tank weapons, they are equipped with a gyroscopic stabilization device. The key element of this device is the gyroscope[5].

When talking about the importance of the gyroscope in the automatic control system of a rocket, the gyroscope is used to measure the angle of deviation of the rocket axis from the initially set direction. That is, initially the gyroscope axis and the direction of the rocket are set in such a way that they overlap, and if for some reason the direction of the rocket changes, an angle of deviation arises between the direction of the rocket and the gyroscope axis. The automatic system, linked to the gyroscope, promptly notifies the automatic control mechanism, which, taking this into account, automatically corrects the direction of movement.

As an example of the use of a gyroscope, consider a projectile rotating around its axis of symmetry. Due to the oscillating motion of the barrel and the pressure difference between the rear and front of the projectile, the projectile's axis forms a specific angle with its trajectory. Immediately after leaving the barrel, the projectile moves in a nearly straight line. If the projectile is tilted upward immediately after leaving the barrel, air pressure will be exerted on its rear. At this point, the warhead is deflected to the right by the gyroscopic effect. In this case, air pressure deflects it to the left, causing the projectile to tilt downward. This occurs because, under the influence of constant air pressure, the projectile's nose moves in a spiral around the direction of its trajectory. This movement of the projectile is called procession. It is precisely this rotation that maintains the projectile's axis in its trajectory. As a result, it moves along a symmetrical parabola and hits the target. If it did not perform a circular motion, the gyroscopic effect would not be observed, and the trajectory would not be parabolic, causing it to fall closer to the target without actually hitting it [6,7,8,9].

A compass is crucial for navigating a ship sailing across the vast ocean, at sea, or an airplane flying in similar conditions; they navigate their destinations solely by the readings of this compass. If such compasses are affected by an abnormal magnetic field in a particular location and show the wrong direction, they will certainly be lost. Most of the losses in the Bermuda Triangle were also due to this reason. If a gyrocompass were used instead of such compasses, such anomalous magnetic fields would not affect them, and losses would certainly be reduced. This is because the main mechanism of such a compass is a gyroscope, which is not affected by magnetic fields[10].

In conclusion, the following can be said:

Any machines and mechanisms, even if their parts are made mainly of solid materials, their movement is explained by the laws of mechanics of deformable solids.

In solid mechanics, there is the concept of a gyroscope and its effects, which is used in almost all technical devices, even as an automatic device.

Today's world is turbulent, and military conflicts continue due to various disagreements. Unmanned aerial vehicles and drones are coming to the rescue. Gyroscopes are also effectively used in unmanned aerial vehicles and countermeasures.

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