INTEGRALLARNI HISOBLASHNING BA'ZI INNOVATSION USULLARI

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Annotatsiya
Matematik analiz kursida aniq va aniqmas integrallar kata o'rin egallaydi.Ularga oid misollarni yechishda talabalar tez-tez qiyinchiliklarga duch keladilar.Bunda ularni hisoblash usullarini tanlash muhim o'rin tutadi.

Kalit so'zlar. Определённый и неопределённый интегралы, методы вычисления интегралов, метод интегрирование по частям, метод замена переменного,понятие редукция,инновационный способ.

SOME INNOVATIVE WAYS OF CALCULATING INTEGRALS

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Annotation
Definite and indefinite integrals occupy a large place in the course of mathematical analysis. Students often encounter difficulties when solving examples related to them. The choice of methods for calculating them plays an important role in this.

Keywords. Definite and indefinite integrals, methods for calculating integrals, piecemeal integration method, variable substitution method, the concept of reduction, innovative method.

НЕКОТОРЫЕ ИННОВАЦИОННЫЕ СПОСОБЫ ВЫЧИСЛЕНИЯ ИНТЕГРАЛОВ

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Аннотация
Определённые и неопределенные интегралы занимают большое место в курсе математического анализа. Учащиеся часто сталкиваются с трудностями при решении связанных с ними примеров. Немаловажное место в этом занимает выбор методов их вычисления.

Ключевые слова: Определенные и неопределенные интегралы, методы вычисления интегралов, метод пошагового интегрирования, метод замены переменных, концепция редукции, инновационный метод.

Ma'lumki talabalar amaliyot darslarida misollar va masalalar yechishda qiyna-ladilar.Ular doimo uslubiy yordamga muhtoj bo'ladilar.Shu maqsadda ushbu maqolada integrallarni yechishning ikkita innovatsion usuli haqida gapiriladi. Quyida integrallarni algebraic tenglamaga keltirib yechish va reduksion formu-lalardan foydalanish usullalari keltiriladi. Dastlab birinchi usul-algebraiik tenglamaga keltirib yechish usulini qaraymiz.

1-Misol. $\int \sqrt{1-x^2}\,dx$ integralni hisoblang.

Yechiish.Agar ushbu integralda $u=\sqrt{1-x^2}, dv=dx, du=\frac{-xdx}{\sqrt{1-x^2}}, v=x$ deb belgilasak u quyidagi ko'rinishga keladi

$$\int \sqrt{1-x^2}\,dx=\sqrt{1-x^2}\cdot x-\int x\cdot\frac{-xdx}{\sqrt{1-x^2}}=x\cdot\sqrt{1-x^2}-\int\frac{(1-x^2-1)dx}{\sqrt{1-x^2}}=x\cdot\sqrt{1-x^2}-$$

$$-\int \sqrt{1-x^2}\,dx+\int\frac{dx}{\sqrt{1-x^2}}$$

Agar bu yerda $\int \sqrt{1-x^2}\,dx=I$ deb belgilasak oxirgi integralni

$$I=x\cdot\sqrt{1-x^2}-I+\int\frac{dx}{\sqrt{1-x^2}}=x\cdot\sqrt{1-x^2}-I=\arcsin x+C$$

yoki

$$2I=x\cdot\sqrt{1-x^2}+\arcsin x+C$$

ko'rinishda yozish mumkin. Bundan esa

$$I=\frac{1}{2}x\cdot\sqrt{1-x^2}+\frac{1}{2}\arcsin x+C$$

ntijani olamiz.

2-Misol. $\int e^{ax}\sin bxdx$ integralni hisoblang.

Yechiish.Agar ushbu integralda $u=e^{ax}, dv=\sin bxdx, du=ae^{ax}, v=-\frac{1}{b}\cos bx$ deb belgilasak u quyidagi ko'rinishga keladi

$$I=\int e^{ax}\sin bxdx=e^{ax}\left(-\frac{1}{b}\cos bx\right)-\int\left(-\frac{1}{b}\cos bx\right)ae^{ax}dx$$

Bu integralni yana bir marta bo'laklab integrallaymiz

$$I=-e^{ax}\frac{1}{b}\cos bx+\frac{a}{b}\int\cos bxe^{ax}dx=\left|\begin{matrix}u=e^{ax},du=ae^{bx}\\ dv=\cos bxdx, v=\frac{1}{b}\sin bx\end{matrix}\right|=-e^{ax}\frac{1}{b}\cos bx+e^{bx}\frac{a}{b^2}\sin bx+$$

$$+\frac{a^2}{b^2}\int\sin bxe^{ax}dx$$

bundan esa

$$I=-e^{ax}\frac{1}{b}\cos bx+e^{bx}\frac{a}{b^2}\sin bx-\frac{a^2}{b^2}I$$

tenglamani olamiz. Uni yechib natijada quyidagilarni hosil qilamiz

$$I+\frac{a^2}{b^2}I=-e^{ax}\frac{1}{b}\cos bx+e^{bx}\frac{a}{b^2}\sin bx,\ I=\left(\frac{a}{a^2+b^2}\sin bx-\frac{b}{a^2+b^2}\cos bx\right)e^{ax}+C.$$

3-Misol. $\int\sec^3 xdx$ integralni hisoblang. 2

Yechish. DI metoddan foydalanamiz

D	I
$\sec x$	$\sec^2 x$
$\sec x\cdot tgx$	tgx

$$I=\int\sec^3 xdx=\sec x\cdot tgx-\int\sec x\cdot tg^2xdx=\sec x\cdot tgx-\int(\sec^3 x-\sec x)dx.$$

Bundan esa

$$I=\sec x\cdot tgx-I+\int\sec dx=\sec x\cdot tgx+\ln|\sec x+tgx|$$

yoki

$$2I=\sec x\cdot tgx+\int\sec dx=\sec x\cdot tgx+\ln|\sec x+tgx|$$

tenglamani olamiz

$I=\frac{1}{2}\sec x\cdot tgx+\frac{1}{2}\ln|\sec x+tgx|+C.$

Endi ikkinchi usul- reduksion formulalar yordamida yechishni qaraymiz.

1-Quyidagi integral berilgan bo'lsin**.**

$I=\int \sin^n xdx.$

Yechish. Buni quyidagicha yozib olamiz.

$$I_n=\int \sin^n xdx=\int \sin^{n-1} x\sin xdx=\left|\begin{matrix}u=\sin^{n-1}x, du=(n-1)\sin^{n-2}xdx\\ dv=\sin xdx, v=-\cos x\end{matrix}\right|=\sin^{n-1}x(-\cos x)-$$

$$\int (n-1)\sin^{n-2}x\cos x(-\cos x)dx=\sin^{n-1}x(-\cos x)+(n-1)\int \sin^{n-2}x\cos^2 xdx=$$

$$=\sin^{n-1}x(-\cos x)+(n-1)\int \sin^{n-2}x\cos^2 xdx=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}x(1-\sin^2 x)dx=$$

$$=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}xdx-(n-1)\int \sin^n xdx=-\sin^{n-1}x\cos x+(n-1)I_{n-2}-(n-1)I_n.$$

Bundan esa $I_n+(n-1)I_n=-\sin^{n-1}x\cos x+(n-1)I_{n-2}-(n-1)I_n$

yoki quyidagi rekurrent formulani olamiz

$$I_n=-\frac{1}{n}\sin^{n-1}x\cos x+\frac{n-1}{n}I_{n-2}. \quad (1)$$

Quyidagi hususiy hollarni qaraymiz.

4-Misol. Quyidagi integral berilgan bo'lsin**.**

$I=\int \sin^5 xdx.$

Yechish. Bizda $n=5$ holda $I_5=\int \sin^5 xdx$ ni (1) formulaga asosan

$$I_5=\int \sin^5 xdx=\frac{1}{5}\sin^4 x\cos x+\frac{5-1}{5}\int \sin^3 xdx=\frac{1}{5}\sin^4 x\cos x+\frac{4}{5}I_3$$

ni yozamiz. Endi dastlab $n=1$ bo'lganida $I_1=\int \sin xdx=-\cos x+C,$

$n=2$ bo'lganida $I_2=\int \sin^2 xdx=\frac{1}{2}\int (1-\cos 2x)dx=\frac{1}{2}x-\frac{1}{4}\sin 2x+C,$ $n=3$ bo'lganida

$$I_3=\int \sin^3 xdx=-\frac{1}{3}\sin^2 x\cos x+\frac{3-1}{3}I_1+C=-\frac{1}{3}\sin^2 x\cos x+\frac{2}{3}I_1+C=$$

$$=-\frac{1}{3}\sin^2 x\cos x-\frac{2}{3}\cos x+C$$

Bo'lgani sababli

$$I_5=\int\sin^5 xdx=\frac{1}{5}\sin^4 x\cos x+\frac{5-1}{5}\int\sin^3 xdx=\frac{1}{5}\sin^4 x\cos x-\frac{4}{15}\sin^2 x\cos x-\frac{4}{9}\cos x+C$$

natijani hosil qilamiz.

2-formula. Quyidagi integralni hisoblang**.**

$$I=\int\cos^n xdx.$$

Yechish: Xuddi yuqoridagi misoldagi kabi

$$I_n=-\frac{1}{n}\sin^{n-1} x\cos x+\frac{n-1}{n}I_{n-2}.$$

Yuqoridagi ko'rinishdagi integrallar bilan birgalikda yana

$$I=\int\frac{dx}{\cos^n x}dx \text{ va } I=\int\frac{dx}{\sin^n x}dx$$

ko'rinishdagi integrallarni ham qarash mumkin.

5-Misol. Hisoblang $I=\int\frac{dx}{\sin^3 x}dx$

Yechish. Bo'laklab integrallash formulasidan foydalanamiz

$$I=\int\frac{dx}{\sin^3 x}dx=\left|\begin{array}{l}\mathrm{u}=\frac{1}{\sin x},\ d\mathrm{u}=-\frac{1}{\sin^2 x}\cos x\frac{1}{\sin x}dx\\ d\mathrm{v}=d(\mathrm{ctgx}),\ \mathrm{v}=\mathrm{ctgx}\end{array}\right|=-\left(\frac{1}{\sin x}\mathrm{ctgx}-\int\frac{1}{\sin^2 x}\cos x\right)\mathrm{ctgx}dx=$$

$$=-\frac{\mathrm{ctgx}}{\sin x}-\int\frac{1}{\sin^2 x}\cos x\,\mathrm{ctgx}dx=-\frac{\mathrm{ctgx}}{\sin x}-\int\frac{1}{\sin^3 x}\cos^2 xdx=-\frac{\mathrm{ctgx}}{\sin x}-\int\frac{1-\sin^2 x}{\sin^3 x}dx=$$

$$=-\frac{\mathrm{ctgx}}{\sin x}-\int\frac{1}{\sin^3 x}dx-\int\frac{1}{\sin x}dx=-\frac{\mathrm{ctgx}}{\sin x}-I-\int\frac{1}{\sin x}dx.$$

$$I=-\frac{\mathrm{ctgx}}{\sin x}-I-\int\frac{1}{\sin x}dx$$

tenglamadan

$$I=-\frac{1}{2}\frac{\mathrm{ctgx}}{\sin x}-\frac{1}{2}\int\frac{1}{\sin x}dx \quad \text{yoki} \quad I=-\frac{1}{2}\frac{\mathrm{ctgx}}{\sin x}-\frac{1}{2}\ln\left|\mathrm{tg}\frac{x}{2}\right|+C.$$

3-formula. Quyidagi integralni hisoblang $\int x^n e^x \mathrm{dx}$.

Yechish. Bo'laklab integrallash formu

$$I_n = \int x^n e^x \mathrm{dx} = \begin{vmatrix} u = x^n, du = nx^{n-1}dx \\ e^x dx = dv, v = e^x \end{vmatrix} = x^n e^x - n\int x^{n-1} e^x dx = x^n e^x - n\int x^{n-1} e^x dx = x^n e^x - nI_{n-1}.$$

Endi bir nechta xususiy hollarni qaraymiz**.**

$$I_1 = \int xe^x \mathrm{dx} = \begin{vmatrix} u = x, du = dx \\ e^x dx = dv, v = e^x \end{vmatrix} = xe^x - \int e^x dx = xe^x - e^x + C,$$

$$I_2 = \int x^2 e^x \mathrm{dx} = \begin{vmatrix} u = x^2, du = 2xdx \\ e^x dx = dv, v = e^x \end{vmatrix} = x^2 e^x - \int 2xe^x dx = x^2 e^x - 2\int xe^x dx = x^2 e^x - 2I_1,$$

$$I_3 = \int x^3 e^x \mathrm{dx} = \begin{vmatrix} u = x^3, du = 3x^2 dx \\ e^x dx = dv, v = e^x \end{vmatrix} = x^3 e^x - \int 3x^2 e^x dx = x^3 e^x - 3\int xe^x dx = x^3 e^x - 3I_2,$$

5-Misol. $\int x^4 e^x \mathrm{dx}$ integralni hisoblang.

$$I_4 = \int x^4 e^x \mathrm{dx} = \begin{vmatrix} u = x^4, du = 4x^3 dx \\ e^x dx = dv, v = e^x \end{vmatrix} = x^4 e^x - 4\int x^3 e^x dx = x^4 e^x - 4\int x^3 e^x dx = x^4 e^x - 4I_3 =$$

$$= x^4 e^x - 4\left(x^3 e^x - 3I_2\right) = x^4 e^x - 4x^3 e^x + 4.3I_2 = x^4 e^x - 4x^3 e^x + 12(x^2 e^x - 2I_1).$$

Demak berilgan integral

$$\int x^4 e^x \mathrm{dx} = x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24xe^x - 24e^x + C$$

ga teng ekan.

Xulosa

Mavzuga oid misollarni yechishda innovatsion usullardan foydalanish talabalarni amaliy malakalarini shakllantiradi,ulrni ijodiy yondashishga yo'nalti-radi.Yosh avlodning matematik madaniyatini rivojlantiradi.

Foydalanilgan adabiyotlar.

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