PROBLEMS RELATED TO HYPERBOLIC EQUATIONS

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Abstract
The article provides information on problems related to hyperbolic equations.

Keywords: Hyperbolic, typical model, wave equation, Burgers equation, finite differences.

Giperbolik tipdagi hususiy hosilali diffefrensial tenglamalar bilan bog'liq bir nechta tipik model muammolarini sanab o'tamiz.
Adveksiya tenglamasi (bir tomonlama to'lqin tenglamasi):

$$u_t + au_x = 0, \qquad 0 < x < 1,$$
$$u(x,0) = \eta(x), \ IC, \qquad (5.1)$$

$u(0,t) = g_l(t)$ $agar\ a \geq 0,\ yoki$ $u(1,t) = g_r(t)$ agar a≤ 0
Bu yerda g_l va g_r mos ravishda chapdan va o'ngdan chegaraviy shartlari belgilanadi.

- Ikkinchi tartibli chiziqli to'lqin tenglamasi:

$$u_{tt} = au_{xx}, \quad 0 < x < 1,$$
$$u(x,0) = \eta(x), \quad \frac{du}{dt}(x,0) = v(x), \quad \text{IC}, \qquad (5.2)$$
$$u(0,t) = g_l(t), \qquad u(1,t) = g_r(t), \quad BC.$$

- Birinchi tartibli chiziqli giperbolik sistema:

$$u_t = Au_x + f(x,t), \qquad (5.3)$$

bu yerda u va f vektorlar, A matritsa. Agar A matritsaning barcha xos qiymatlari haqiqiy sonlar bo'lib uni $A = TDT^{-1}$ ko'rinishda ifodalab bo'lsa (bu yerda D dioganal matritsa, T determenanti noldan farqli matritsa) (5.3) tenglama giperbolik deb ataladi.

- Nochiziqli giperbolik tenglama yoki sistema, saqlanish qonunlari:

$$u_t + f(u)_x = 0, va\ h.k.\ Byurgers\ tenglamasi\ u_x + (\frac{u^2}{2})_x = 0; \quad (5.4)$$

$$u_t + f_x + g_y = 0. \qquad (5.5)$$

Chiziqli bo'lmagan giperbolik tipdagi hususiy hosilali diffefrensial tenglamalarda boshlanģich berilganlar silliq bólgan vaqtda ham uzlukli(chekli) yechim mavjud bo'ladi.

Xarakteristikalar va chegaraviy shartlar

Biz bir tomonlama to'lqin tenglamasining aniq yechimini bilamiz

$$u_t + au_x = 0, \quad -\propto < x < \propto,$$
$$u(x,0) = \eta(x), \qquad t > 0$$
$$u(x,t) = \eta(x - at)$$

Agar masala qo'yilgan soha chegaralangan bo'lsa, biz aniq yechimni ham topa olamiz.

Quyidagi model muammosini yechim xarakteristikalar bo'yicha o'zgarmas bo'lgani uchun xarakteristikalar usuli bilan hal qilamiz:

$$u_t + au_x = 0, \qquad 0 < x < 1,$$
$$u(x,0) = \eta(x),\ \ t > 0 \quad u(0,t) = g_l(t)\ \ agar\ a > 0\ bo'lsa$$

Har qanday (x; t) nuqta uchun biz yechimni boshlang'ich shartga qarab topishimiz mumkin. Aslida, xarakteristikasi uchun

$$z(s) = u(x + ks, t + s) \qquad (5.6)$$

bo'ylab yechim o'zgarmas $(z'(s) \equiv 0)$, buni hususiy hosilali diffefrensial tenglamaga qo'yib biz quyidagiga ega bo'lamiz:

$$z'(s) = u_t + ku_x = 0,$$

k = a bo'lsa bu tenglik har doim o'rinli bo'ladi.

$(x + ks, t + s)$ dagi yechim (x; t) nuqtadagi yechim bilan bir xil, shuning uchun chiziq to'g'ri kelguncha orqaga qarab chegara chizig'igacha masalani yechishimiz mumkin.

Chegara, ya'ni $u(\bar{x},\bar{t}) = u(x + as, t + s) = u(x - at, 0)\ agar\ x - at \geq 0$ bo’lsa, dastlabki holatga qaytamiz. Agar x –at < 0 bo’lsa, biz faqat x = 0 yoki $s = -\frac{\bar{x}}{a}\ \ va\ t = \frac{\bar{x}}{a}$, va yechim $u(\bar{x},\bar{t}) = u\left(0, t - \frac{\bar{x}}{a}\right) = g_l\left(t - \frac{\bar{x}}{a}\right)$.

Shuning uchun a ≥ 0 holining yechimini quyidagicha yozish mumkin:

$$u(x,t) = \begin{cases} \eta(x - at) agar\ x \geq at\ bo'lsa \\ g_l\left(t - \frac{x}{a}\right) agar\ x \geq at\ bo'lsa \end{cases} \qquad (5.7)$$

Endi biz nima uchun x = 0 da chegara shartini belgilashimiz kerakligini ko’ramiz, lekin biz x = 1 da hech qanday chegaraviy shartga ega bo’lmaymiz. Giperbolik masalalar uchun chegara shartlarini to’g’ri tanlash juda muhim! Bir o’lchamli to’lqin tenglamasi ko’pincha giperbolik masalalar uchun turli xil sonli metodlarni sinovdan o’tkazish uchun namunaviy masala sifatida ishlatiladi.

Chekli ayirma sxemalar.

Giperbolik tenglamalarning oddiy sonli usullar quyidagilarni o’z ichiga oladi:

• Laks-Fridrix usuli;
• Shamolga qarshi harakat sxemasi;
• Leap-qurbaqa usuli (esda tuting, u issiqlik tenglamasi uchun ishlamaydi, lekin chiziqli giperbolik tenglamalar uchun ishlaydi);
• Quti sxemasi;
• Laks-Vendroff usuli;
• Krank – Nikolson sxemasi (giperbolik turdagi masalalar uchun tavsiya etilmaydi, chunki vaqt qadami uchun cheklovlar yo’q); va
• Nur–Isitish usuli (agarda yechim silliq bo’lsa bir tomonlama ikkinchi tartibli shamolga qarshi sxemasi).

Shuningdek, yuqori tartibli metodlar ham mavjud. Agar dastlabki ma'lumotlar silliq bo’lsa (uzilishlar bo’lmasa), chiziqli giperbolik masalalar uchun Laks-Vendroff metodiga o’xshash ikkinchi tartibli aniq metodlarni qo’llash tavsiya etiladi. Biroq, boshlang’ich qiymat chekli uzilishlarni o’z ichiga olsa, ehtiyot bo’lish kerak, chunki ikkinchi yoki yuqori tartibli metodlar ko’pincha uzilishlar yaqinida tebranishlarga olib keladi (Gibbs hodisalari). Ba'zi metodlar saqlanish qonuniga oid sonli metodlarning asosini tashkil qiladi, bu maxsus konservativ, noaniq giperbolik tizim bo'lib, bu tizimda uzilishlar boshlang'ich berilganlar silliq bo'lsa ham, chekli vaqt ichida rivojlanishi mumkin. Shuningdek, giperbolik differensial tenglamalari uchun, odatda, parabolik turdagi masalalar uchun bo'lgani kabi qat'iy bir vaqt qadami cheklovlari mavjud emas. Ko'pincha aniq metodlar afzal ko'riladi.

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