# DIFFERENCE EQUATIONS

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**Abstract.** The article provides information about difference equations.

**Keywords:** Differential equation, difference equation, right difference, left difference, central difference, approximate value.

Differensial tenglamalar bilan ifodalanivchi texnik va matematik masalalar mavjud. Bunday masalalarni ayirmali usullar bilan yechish ayirmali tenglamalarga olib keladi.

Arifmetik progressiya formulasi hadlari uchun o'rinli bo'lgan $a_{k+1} = a_k + d$ yoki $a_{k-1} - 2a_k + a_{k+1} = 0$ tenglamalar ayirmali tenglamalardir. Bu yerda $a_k = a(k),\ k = 1,2,3\ldots$

Ya'ni, argument k butun qiymatlarini qabul qiladi.

Endi argumant butun qiymatlarini qabul qiluvchi funksiyani qaraymiz

$$y(i), \quad i = 0, \pm 1, \pm 2, \ldots$$

i nuqtada quyidagi ayirmani yozamiz:

O'ng ayirma $\Delta y_i = y(i+1) - y(i)$,

Chap ayirma $\nabla y_i = y(i) - y(i-1)$.

Odatda $y_i = y(i)$ belgilash qabul qilingan. U holda

$$\Delta y_i = y_{i+1} - y_i\,, \qquad \nabla y_i = y_i - y_{i-1}\,.$$

Bu ifodalarni birinchi tartibli hosilani formal analogi sifatida qarash mumkin. Ikkinchi tartibli ayirmani qaraymiz

$$\Delta^2 y_i = \Delta(\Delta y_i) = \Delta(y_{i+1} - y_i) = (y_{i+2} - y_{i+1}) - (y_{i+1} - y_i) = y_{i+2} - 2y_{i+1} + y_i$$

$\Delta y_{i-1} = \nabla y_i$ ekanligini qatd etish lozim. Hqiqtdan ham tenglikning har ikki tomoni ham $y_i - y_{i-1}$ ga teng.

Chap ayirmali operatorni qo'llash o'ng ayirmali operatorni bir birlik chap nuqtaga qo'llash bilan teng kuchli, ya'ni

$$\Delta\nabla y_i = \Delta^2 y_{i-1} = y_{i+1} - 2y_i + y_{i-1}$$

Xuddi shuningdek, $\Delta^m y_i$ aniqlanadi

$$\Delta^m y_i = \Delta(\Delta^{m-1} y_i)$$

$\Delta$ operatorini har bir marta qo'llaganda o'ng tomonidan yana bitta nuqta ayirmali ifodada ishtirok etadi. $\Delta$ operatorni m marta qo'llab, funksiyaning $i, i+1, \dots, i+m$ nuqtalardagi $y_i, y_{i+1}, \dots, y_{i+m}$ qiymatlaridan tashkil topgan $\Delta^m y_i$ ni hosil qilish mumkin

Turli tartibli ayirmalar ishtirok etuvchi ayirmali tenglamani quyidagicha yozish mumkin:

$$\alpha_0 \Delta^m y_i + \alpha_1 \Delta^{m-1} y_i + \dots + \alpha_{m-1} \Delta y_i + \alpha_m y_i = f_i$$

Bu yerda $\alpha_0, \alpha_1, \dots, \alpha_m$ koeffisientlar bo'lib, $\alpha \neq 0$. Bu tenglama butun argumentning funksiyasi bo'lib nomalum funksiya - $y_i$ ga nisbatan $m$ −chi tartibli ayirmali tenglama deyiladi.Bu ayirmali tenglama $m$ − taribli quyidagi

$$\alpha_0 \frac{d^m u}{dx^m} + \alpha_1 \frac{d^{m-1} u}{dx^{m-1}} + \dots + \alpha_{m-1} \frac{du}{dx} + \alpha^m u = f\,, \qquad \alpha_0 \neq 0.$$

Differensial tenglamaning analogidir. Differensial tenglamaning koeffisientlari x argumentning funksiyasi bo'lgani kabi ayirmali tenglamaning koeffsientlari $\alpha_m = \alpha_m(i)$ lar I ga bog'liq bo'ladi.

Oddiy differensial tenglamaga sodda misol keltiramiz.

$$\frac{du}{dx} = f(x)$$

Differensial tenglamani yechish talab etilgan bo'lsin. Bu tenglamadagi hosilani taqriban ayirmali ifoda bilan almashtirish mumkin:

$$\left(\frac{du}{dx}\right)_{x=x_i} : \frac{u(x_i+h) - u(x_i)}{h} \;,$$

bu yerda $h > 0$ $x_i$ va $x_i + h$ nuqtalar orasidagi masofa. Agar $x_i + h = x_{i+1}$

$u(x_i) = u_i,\ u(x_{i+1}) = u_{i+1}$ belgilashlar keltirsak, u holda

$$\left(\frac{du}{dx}\right)_{x=x_i} : \frac{\Delta u_i}{h} = \frac{u_i - u_{i-1}}{h} \;.$$

O'ng va chap ayirmalarni yig'indisining yarmi markaziy ayirmani ifodalaydi:

$\left(\frac{du}{dx}\right)_{x=x_i} : \frac{\Delta u_i + \nabla u_i}{2h} = \frac{u_{i+1} - u_{i-1}}{2h}$ .

Hamma yerda : belgisi moslik yoki approksimatsiyani bildiradi. $\frac{\Delta u_i}{h} = \frac{u_{i+1} - u_i}{h}$ ifoda $\frac{du}{dx}$ hosilani approksimatsiya qiladi deyiladi.

Shunday qilib,

$$\frac{\Delta y_i}{h} = f_i \,, \qquad f_i = f(x_i)$$

tenglamani qaraymiz. Ta'rifga ko'ra bu birinchi tartibli ayirmali tenglamalardir. Uni quyidagicha yozish mumkin

$\Delta y_i = hf_i$ yoki $y_{i+1} = y_i + hf_i$ .

Birinchi tartibli differensial tenglamani almashtirishda ikkinchi tartibli ayirmali tenglamaga ham ega bo'lish mumkin. Masalan,

$$u(x_{i+1}) = u(x_i) + hu'(x_i) + 0.5h^2 u''(x_i) + \frac{h^3}{6} u'''(x_i) + O(h^4),$$

$$u(x_{i-1}) = u(x_i) - hu'(x_i) + 0.5h^2 u''(x_i) - \frac{h^3}{6} u'''(x_i) + O(h^4).$$

Bu ikki ifoda qo'shilib,

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} = u_i'' + O(h^2)$$

Ifodaga ega bo'lish mumkin. Bu yerda $O(h^2)$ ni tashlab yuborib, $u_i''$ uchun taqribiy

$$\left(\frac{du^2}{dx^2}\right)_{x=x_i} : \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} = \frac{\Delta^2 u_{i-1}}{h^2} = \frac{\Delta \nabla u_i}{h^2}$$

ifoda hosil qilish mumkin.

$u_{i+1}$ ni $x_i$ nuqta atrofida Teylor qatoriga yoyilmasi $u_{i+1} = u_i + hu_i' + 0{,}5h^2 u_i'' + O(h^3)$ da $u_i''$ ikkinchi tartibli ayirmali ifoda bilan almashtirib,

$$u_i' = \frac{u_{i+1} - u_i}{h} - \frac{h}{2} \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + O(h^2)$$

Ya'ni ikkinchi tartibli ayirmali tenglamani hosil qilish mumkin.Buni ikkinchi tartibli ayirmali tenglama ekanliginiquyidagicha isbotlash mumkin. Oxirgi formulada $u_i$ $ni$ $f_i$ ga almashtirib, $O(h^2)$ hadni tashlab yuborib, hosil bo'lgan tenglamani 2h ga ko'paytiramiz. U holda birinchi tartibli differensial tenglama $\frac{du}{dx} = f$ o'rniga quyidagi ikkinchi tartibli ayirmali tenglamaga ega bo'lamiz

$$\Delta \nabla y_i - 2\Delta y_i = 2hf_i \,.$$

**Foydalanilgan adabiyotlar**

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