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ANALYSIS OF MAGNETIC FLUX DISTRIBUTION IN NONLINEAR MAGNETIC CIRCUITS WITH DISTRIBUTED PARAMETERS AND A MOVABLE CORE USING THE RUNGE–KUTTA METHOD

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ABSTRACT:
The article presents the nonlinear differential equations characterizing the parameters of the processes in a nonlinear magnetic circuit with a distributed parameter and a movable core. These equations are solved using the Runge–Kutta method. The obtained solutions are compared with experimentally measured values, and the relative errors are determined. Considering the small magnitude of the errors and the fact that exact analytical solutions for nonlinear differential equations do not exist, it is proven that applying the numerical Runge–Kutta method is an effective approach to finding solutions.

KEYWORDS: Magnetic circuit, induction, magnetic flux, copper core, magnetic core, EMF (electromotive force), MMF (magnetomotive force), magnetic potential.

INTRODUCTION

Nonlinear magnetic circuits with distributed parameters and a movable core are widely used as a main element in measurement transducers applied in technological processes. Taking into account the nonlinearity of magnetic circuits and accurately expressing the quantities that characterize them increases the accuracy of measurement transducers. Therefore, in this article, the characteristic quantities of the magnetic circuit are calculated while considering the nonlinearity. Figure 1 shows the structural diagram of a nonlinear magnetic circuit with distributed parameters and a movable core. In this configuration, the movable core moves along the coordinates x_1 and x_2, converting displacement motion into EMF in the measuring coil. The excitation coil is supplied with an EMF denoted as Eq, which generates a constant magnetic flux along the excitation magnetic core of the magnetic circuit. Due to magnetic permeability and the movable core, a varying magnetic flux is generated in the magnetic core wound with the measuring coil[1,2,3].

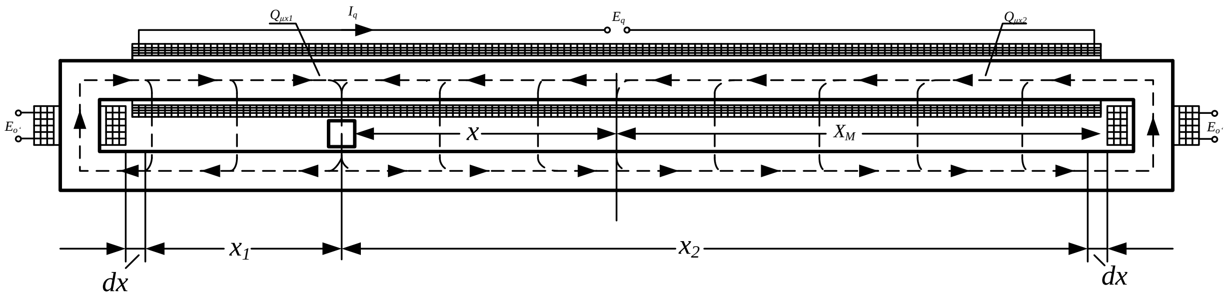

Figure 1. The structural diagram of a Distributed Parameter Nonlinear Magnetic Circuit (DPNMC) with a movable core is presented.

The quantities characterizing the processes in distributed-parameter nonlinear magnetic circuits with a movable and shielded part are governed by second-order differential equations, and the problem of finding the solution to these equations is considered.

We derive equations for the left and right sides of the magnetic circuit for an element of length dx based on Ohm's and Kirchhoff's first and second laws for magnetic circuits[2,5,6,7,8].

$$\frac{dQ_{\mu x_1}}{dx_1} = C_{\mu p} U_{\mu x_1} \quad (1) \qquad \frac{dQ_{\mu x_2}}{dx_2} = C_{\mu p} U_{\mu x_2} \quad (4)$$

$$\frac{dU_{\mu x_1}}{dx_1} = 2Z_{\mu p} Q_{\mu x_1} \quad (2) \qquad \frac{dU_{\mu x_2}}{dx_2} = 2Z_{\mu p} Q_{\mu x_2} \quad (5)$$

$$Z_{\mu x_1} = \frac{U_{\mu x_1}}{Q_{\mu x_1}} \quad (3) \qquad Z_{\mu x_2} = \frac{U_{\mu x_2}}{Q_{\mu x_2}} \quad (6)$$

Here: $C_{\mu p}$ – the magnetic permeance between the magnetic conductors for the element dx of the magnetic circuit; $Z_{\mu p}$ – the magnetic reluctance of the magnetic conductor for the element dx of the magnetic circuit; $Q_{\mu x_1}$ and $Q_{\mu x_2}$ – the magnetic flux varying with respect to coordinates x_1 and x_2; $U_{\mu x_1}$ and $U_{\mu x_2}$ – the magnetic potential varying with respect to coordinates x_1 and x_2; $Z_{\mu x_1}$ and $Z_{\mu x_2}$ – the magnetic reluctance varying with respect to coordinates x_1 and x_2.

By transforming the derived differential equations, we determine the variables that change along the coordinates x_1 and x_2. First, we take the second derivative of the equations characterizing the left side of the magnetic circuit[4].

$$\frac{d^2Q_{\mu x_1}}{dx_1^2} = C_{\mu p} \cdot \frac{dU_{\mu x_1}}{dx_1} \; yoki \; Q''_{\mu x_1} = C_{\mu p} \cdot U'_{\mu x_1} \tag{7}$$

$$\frac{d^2U_{\mu x_1}}{dx_1^2} = 2Z_{\mu p} \cdot \frac{dQ_{\mu x_1}}{dx_1} \; yoki \; U''_{\mu x_1} = 2C_{\mu p} \cdot Q'_{\mu x_1} \tag{8}$$

We substitute expressions (2) and (1) into expressions (7) and (8).

$$\frac{d^2Q_{\mu x_1}}{dx_1^2} = 2C_{\mu p}Z_{\mu p}Q_{\mu x_1} \; yoki \; Q''_{\mu x_1} = 2C_{\mu p}Z_{\mu p}Q_{\mu x_1} \tag{9}$$

$$\frac{d^2U_{\mu x_1}}{dx_1^2} = 2C_{\mu p}Z_{\mu p}U_{\mu x_1} \; yoki \; U''_{\mu x_1} = 2C_{\mu p}Z_{\mu p}U_{\mu x_1} \tag{10}$$

To account for the nonlinearity of magnetic reluctance, the main magnetization curve of the ferromagnetic core made of electrotechnical steel is approximated by an analytical function.

$$H = \alpha sh(\beta B) \tag{11}$$

Here, α and β are the approximation coefficients.

We rewrite expression (9) and present expression (11) as follows:

$$Q''_{\mu x_1} = 2C_{\mu p}\alpha sinh\left(\frac{\beta Q_{\mu x_1}}{S}\right) \tag{12}$$

Since an analytical solution to equation (12), which is a nonlinear second-order differential equation, does not exist, we solve it using one of the numerical methods — the Runge–Kutta method.

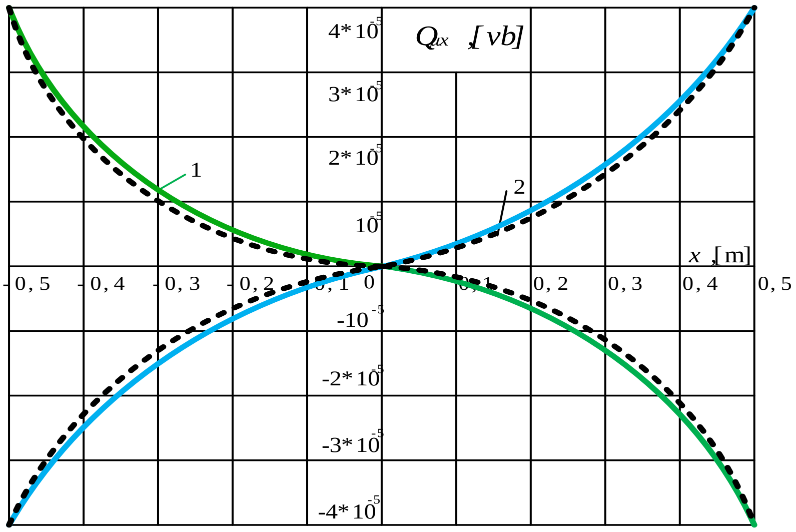

Figure 2. Variation of the magnetic flux in the core as a result of the movement of the movable part.

III. CONCLUSION

When the second-order nonlinear differential equation describing the variation of magnetic flux in a distributed-parameter nonlinear magnetic circuit with a movable core is solved numerically using the 4th-order Runge–Kutta method, the error at specific points compared to experimental values was found to range from 1.2% to 5.6%, with an average relative error of 3.4%. These errors are primarily due to magnetic saturation effects in the magnetic circuit and the relative inaccuracies of the measuring instruments. Since the discrepancy is not significant, the obtained solution to the differential equation can be considered accurate.

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