MATHEMATICAL MODEL OF THE INTERACTION BETWEEN SURFACE WATER AND GROUNDWATER

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Abstract

The article proposes hydrodynamic models describing the interaction between surface water and groundwater under furrow irrigation of cotton. These models allow for characterization of filtration processes and surface runoff at various levels of detail and accuracy. Based on mass-transfer models of the interaction between filtration and surface flows, the processes of water movement in the furrow, infiltration flow within the aeration zone, and variations in the groundwater level within the saturated zone were investigated.

Keywords: Furrow irrigation, filtration, groundwater, turbulent diffusion, mass-transfer process, diffusion coefficient, hydrodynamic parameters, boundary conditions.

Introduction

Boundary Criteria for the Interaction Parameters of Surface and Infiltration Waters

The water regime of irrigated lands is determined by the ratio between the saturation of the active soil layer through atmospheric precipitation, surface irrigation, the removal of moisture through collector–drainage systems, filtration, and evaporation processes. During furrow irrigation, most of the water flow infiltrates into the active soil layer where cotton is planted. In this layer, the highest porosity and hydraulic conductivity are observed, and the groundwater level fluctuates accordingly.

The filtration flow emerging from irrigated fields forms within the active layer, penetrates into adjacent zones, saturates upper horizons, and increases their moisture content [1,5,6,7]. For irrigated areas, modeling the transport of mineralized, mineral, and organic substances through infiltration moisture and evaluating the influence of the filtration component on groundwater quality is an important scientific and technical problem.

Research Methodology

The methodological basis of the study relies on a systems approach, since the research object is a complex, integrated ecological system that functions under the combined influence of natural and anthropogenic factors.

Research Results

We examine the interaction models of surface water and groundwater under furrow irrigation of cotton on irrigated lands, which allow characterization of mass-exchange processes through interacting surface and subsurface flows.

According to the theory of turbulent diffusion, the distribution of mineral and organic elements in the form of concentration of certain substances is governed by transport flow and diffusion flow, which occur due to turbulent fluctuations in transport velocity. The molecular flow can be simplified by considering the thermal motion of molecules [2,3,4,10]. During mass-transfer processes, a substance may undergo changes by entering physical and chemical interactions with other components that alter the mechanical, physical, and chemical properties of particles and substances in the environment [8,9,11].

The quantitative aspect of temporal and spatial variations in the composition of mineral and organic substances in the soil is described by the transfer equation [1].

$$\frac{\partial \varphi}{\partial t}+\frac{\partial (u\varphi)}{\partial x}+\frac{\partial (\upsilon\varphi)}{\partial y}+\frac{\partial (\omega\varphi)}{\partial z}+\sigma\varphi-\mu(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2})-\nu\frac{\partial^2 \varphi}{\partial z^2}=f(x,y,z,t)$$

Here $\varphi(x,y,z,t)$ – the intensity (concentration) of mass exchange that is transported together with the medium (moisture and other substances) or with the airflow;

ν — the horizontal and vertical diffusion coefficients, respectively.;

σ - the absorption coefficient (the ratio of the time interval during which the concentration of the substance decreases by a factor of e relative to the initial intensity);

f — a function describing the internal sources of mass exchange.

At present, new requirements have emerged for hydrodynamic methods of studying turbulent diffusion. In the soils of irrigated lands, there is a need to investigate turbulent flows, the large-scale distribution of mixtures entering from various sources, and to account for numerous parameters. Under the influence of different factors, the thickness of the soil–ground layer in which mixture dispersion occurs, as well as the changes in the structural components of filtration velocity and exchange coefficients within such a layer, become highly complex. Therefore, the actual distribution of all mixtures across the dispersion layer must be considered [5,6].

In the soil–ground moisture zone, filtration velocity and temperature behave as logarithmic functions of depth, while the coefficient of turbulent mass transfer changes with depth according to an exponential law (either increasing or decreasing).

Problems associated with the non-uniformity of the soil–ground layer with depth lead to the necessity of developing models that describe the soil as a medium consisting of several layers. In each layer, physical parameters vary relatively little and may be treated as approximately constant. This allows us to use equations with constant coefficients when formulating the problem.

Let us consider the state of mass exchange of a substance in the soil–ground layer, which is divided into N sublayers with respect to depth. Each sublayer has its own specific values of infiltration flow and turbulent diffusion coefficients. Under the influence of infiltration irrigation water, the processes of advection, turbulent mixing, and mass transfer are taken into account.

Let us assume that there is a constant-layer mass-transfer source (X_0, Y_0, Z_0) located at an arbitrary point of the i-th layer with coordinate z_i.

We assume that the process is steady with respect to time ,H_I-$(I=1,2,\dots,N)$ the thicknesses of the layers are not equal to one another, and we assume that the infiltration source is located at an arbitrary point within the i-th layer. In this case, the differential equation describing the mass-transfer process takes the following form:

$$u_n\frac{\partial\varphi_n}{\partial x}+\upsilon_n\frac{\partial\varphi_n}{\partial y}+(\omega_n-\omega_{gn})\frac{\partial\varphi_n}{\partial z}+\sigma_0\varphi_n-\nu_n\frac{\partial^2\varphi_n}{\partial z^2}-\mu(\frac{\partial^2\varphi_n}{\partial x^2}+\frac{\partial^2\varphi_n}{\partial y^2})=\delta_{ni}C\delta(x-x_0,y-y_0,z-z_0) \quad \textbf{(1)}$$

where C_n is the concentration function of mineral and organic substances in the n-th layer; u_n, v_n, w_n are the components of the filtration velocity in the x, y, and z directions for the n-th layer w_n^* is the absolute value of the vertical velocity caused by gravitational forces in the n-th layer α is the absorption coefficient D_{nz}, D_{nh} are the vertical and horizontal diffusion coefficients for the n-th layer.

$\delta_{ni}=\begin{cases}1, n=1\\0, n\neq 1\end{cases}$ indicates that the source is located in the n-th layer;

$n=1,2,\dots,N$ is the layer index; C is a constant characterizing the hydrodynamic parameters of the infiltration flow. The boundary conditions are formulated as follows:

$$\frac{\partial\varphi_1}{\partial z}-\lambda\varphi_1=q_1,\ z=0,\ \varphi_N=q_2,\ z=h_N \quad (2)$$

Here: q_1 is the thickness of the corresponding field source, usually dependent on coordinates; q_2 is the concentration at the upper boundary of the layer, typically close to zero.

At the boundary, partial absorption and partial reflection of the substance are assumed to occur at $z=0$, and the intensity of interaction with the moisture zone is determined by an empirical coefficient. At $z=h_N$, the upper boundary of the layer, the concentration of mineral substances is assumed to be close to zero. To define the interaction parameters of surface and infiltration waters, as well as the limiting transition criteria from the perspective of moisture in the vadose zone, conditions must be specified at the discontinuity surfaces. We refer to these as **interface conditions**.

The interface conditions at the layer boundaries are expressed as equality of concentration values and the rates of change of concentration at the boundary between two adjacent layers:

$$\begin{cases} \varphi_i = \varphi_{i-1} \\ \qquad\qquad i = 2,3.....N-1, z = h_{i-1} \\ \dfrac{\partial \varphi_1}{\partial z} = \dfrac{\partial \varphi_{i-1}}{\partial z} \end{cases} \tag{3}$$

Equations (1)–(3) represent a fairly general formulation of the problem. The method applied within the scope of this article consists of applying integral transforms, followed by numerical analysis of the resulting relationships [2,3,4]. One of the conveniences provided by the integral transform method is the ability to formulate mixed boundary-value problems in terms of integral equations. This approach allows the analytical construction of the Fourier transform solution for the given problem.

By applying the two-dimensional Fourier transform to the initial equation, boundary conditions, and interface conditions, the differential equation can be reduced to a quasi-linear differential equation:

$$\frac{d\overline{\varphi}_n}{dz^2} - \frac{\omega_n - \omega_{gn}}{v_n}\frac{d\overline{\varphi}_n}{dz} - \frac{1}{v_n}\left[-i(au_n + \beta u_n) + \mu_n(a^2 + \beta^2) + \sigma_0\right]\overline{\varphi}_n = -\delta_{ni}\frac{C}{v_n}\exp(i(ax_0 + \beta y_0))\delta(z - z_0) \qquad \textbf{(4)}$$

In this case, the boundary conditions take the following form:

$$\frac{d\overline{\varphi}_1}{dz} - \lambda\overline{\varphi}_1 = q_1, z = 0, \overline{\varphi}_N = q_2\,, \; z = h_N \tag{5}$$

The interface (continuity) conditions take the following form:

$$z = h_{i-1}\,, \frac{d\overline{\varphi}_i}{dz} = \frac{d\overline{\varphi}_{i-1}}{dz}, \overline{\varphi}_i = \overline{\varphi}_{i-1}, \tag{6}$$

Subsequently, the inverse Fourier transform is applied to the solution obtained in the transform domain.

$$\varphi_n(x,y,z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\overline{\varphi}_n(a,\beta,z)e^{-i(ax+\beta y)}\,da\,d\beta \tag{7}$$

As a result of the numerical transformation of the integral, the original solution of problem (7) is obtained.

Equation (4) is non-homogeneous in the i-th layer, but it is homogeneous for n_i. Its solution can be obtained using a method of continuous solutions specially adapted for such problems.

We divide the i-th layer into two sublayers along a plane passing through a point $z_0$parallel to the boundary. The number of layers accordingly increases by one. In this case, equation (4) becomes uniform for any points $z$and z_0. For each sublayer, the solution can be written separately, and the two solutions are then combined, so that the resulting solution of the non-homogeneous initial equation satisfies the constants.

$$\overline{\varphi}_i^+(z_0)=\overline{\varphi}_i^-(z_0),2\left\{\left[\overline{\varphi}_i^+(z_0)\right]'-\left[\overline{\varphi}_i^-(z_0)\right]'\right\}=-F(a,\beta) \qquad (8)$$

Here $F(A, B) = C\ V^{-1} \exp(i\,(\mathrm{ax}_0 + FIY_0))$.

We rewrite the solution for each sublayer as follows:

$$\overline{\varphi}_n(z)=c_{n1}e^{(\sigma_n-\theta_n)z}+c_{n2}e^{(\sigma_n+\theta_n)z}, n=1,\dots\dots N,\ n\neq i$$

$$\overline{\varphi}_i^-(z)=b_{i1}e^{(\sigma_i-\theta_i)z}+b_{i2}e^{(\sigma_i+\theta_i)z},$$

$$\overline{\varphi}_i^+(z)=b_{i3}e^{(\sigma_i-\theta_i)z}+b_{i4}e^{(\sigma_i+\theta_i)z},\ n=i\ . \qquad (9)$$

Here:

$$\theta_n=-0.5v_n^{-1}((\omega_n-\omega_{gn})^2+4v\left[\mu_n(a^2+\beta^2)+i(au_n+\beta\upsilon_n)+\sigma_0\right])^{0.5}$$

and $\sigma_n = 0.5\upsilon_n^{-1}(\omega_n - \omega_{gn}$ For each layer, the solution is calculated individually..

To find the solution of the problem, the constants $c_n^k$and b_n^kin equations (6) and (8), which satisfy conditions (5), (6), and (8), are determined. For N layers, there are $2(N+1)$coefficients and, correspondingly, $2(N+1)$equations.

The structure of the resulting system is quite simple and can be solved using any standard method. After solving the system of equations, the obtained coefficients are substituted into the formulas (9). The true solutions of the problem in (7) are then obtained numerically by solving the integral.

Conclusion

A hydrodynamic linkage model was developed to describe the relationship between water applied through furrow irrigation in a lysimeter ensemble, infiltration flows in the aeration zone, and the dynamics of groundwater levels in the saturated zone. The developed mechanism allows the reconstruction of the solution for any number of soil layers, providing a representation of flow and filtration velocity distribution over depth that closely approximates the actual characteristics.

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