



GROUND STATES FOR THE SOS MODEL WITH COMPETING INTERACTIONS ON CAYLEY TREE

Rashidova O. Sh.

University of Exact and Social Sciences, Tashkent, Uzbekistan;

rashidovao@gmail.com

The cayley tree $\mathfrak{T}^k$ of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, with exactly $k+1$ edges issuing from each vertex. We suppose that $\mathfrak{T}^k = (V, L)$, where V is the set of vertices of $\mathfrak{T}^k$, L is the set of its edges. Two vertices x and y are called

nearest neighbors if there exists an edge $l \in L$ connecting them. We will use the notation $l = \langle x, y \rangle$. A collection of nearest neighbor pairs $\langle x, x_1 \rangle, \langle x_2, x_3 \rangle, \dots, \langle x_{d-1}, y \rangle$ is called a path from x to y . The distance $d(x, y)$ on the Cayley tree is number of edges of the shortest path from x to y .

Let us x a root of the tree $x^0 \in V$ and we set

$$W_n = \{x \in V \mid d(x, x^0) = n\}, \quad V_n = \{x \in W \mid d(x, x^0) \leq n\}$$

For a given $x \in V_n$ we define a set of direct successors of x as follows

$$S(x) = \{y \in W_{n+1} \mid d(x, y) = 1\}.$$

Let $\Psi = \{0, 1, \dots, m\}$ be a nonempty set. A mapping $\sigma_A: A \rightarrow \Psi$ defined on nonempty subset A of the sets of all vertices of Cayley tree, is called *configuration* on A . Usually, Ψ is called a *set of spin values*. We denote $\Omega_A := \Psi^A$, i.e. Ω_A is the set of all configurations on A . By the same way we can define a configuration σ on V and we denote $\Omega := \Psi^V$.

We study ground states for the SOS model on a Cayley tree. For a pair of configurations σ and φ which coincide almost everywhere, i.e., everywhere except finitely many points, we consider the relative Hamiltonian $H(\sigma, \varphi)$ describing the energy differences of the two configurations σ and φ :

$$\begin{aligned} H(\sigma, \varphi) = & -J_1 \sum_{\substack{x, y \in V \\ d(x, y) = 1}} (|\sigma(x) - \sigma(y)|) - |\varphi(x) - \varphi(y)| - \\ & -J_2 \sum_{\substack{x, y \in V \\ d(x, y) = 2}} (|\sigma(x) - \sigma(y)|) - |\varphi(x) - \varphi(y)| \end{aligned} \quad (1)$$

where $(J_1, J_2) \in \mathbb{R}^2$.

Let M be the set of all unit balls with vertices in V . A restriction of a configuration σ to the ball $b \in M$ is a bounded configuration and it is denoted by σ_b .

We define the energy of the configuration σ on b by the following formula

$$U(\sigma_b) = U(\sigma_b, J_1, J_2) = -\frac{1}{2}J_1 \sum_{x,y \in b} |\sigma(x) - \sigma(y)| - J_2 \sum_{\substack{x,y \in b \\ d(x,y)=2}} |\sigma(x) - \sigma(y)|,$$

where $(J_1, J_2) \in \mathbb{R}^2$.

We consider the case $k = 2$. and $m = 3$. It is easy to see that $U(\sigma_b) \in \{U_i : i = \overline{1,10}\}$ for σ_b , where

$-J_1-3J_2$	$\frac{7}{2}J_1-8J_2$	$-4J_1-4J_2$	$-4J_1$	$\frac{5}{2}J_1-9J_2$
$-J_1-4J_2$	$\frac{7}{2}J_1-11J_2$	$-4J_1-5J_2$	$-\frac{1}{2}J_1-2J_2$	$\frac{5}{2}J_1-6J_2$
$-J_1-5J_2$	$\frac{7}{2}J_1-3J_2$	$-4J_1-6J_2$	$-\frac{1}{2}J_1-3J_2$	$\frac{5}{2}J_1-11J_2$
$-J_1-6J_2$	$\frac{7}{2}J_1-6J_2$	$-4J_1-7J_2$	$-\frac{3}{2}J_1-2J_2$	$-\frac{11}{2}J_1-3J_2$
$-2J_1-4J_2$	$\frac{7}{2}J_1-5J_2$	$-4J_1-8J_2$	$-\frac{3}{2}J_1-3J_2$	$-\frac{11}{2}J_1-2J_2$
$-2J_1-5J_2$	$\frac{7}{2}J_1-2J_2$	$-4J_1-9J_2$	$-\frac{3}{2}J_1-5J_2$	$\frac{9}{2}J_1-9J_2$
$-2J_1-6J_2$	$\frac{7}{2}J_1-10J_2$	$-4J_1-10J_2$	$-\frac{3}{2}J_1-6J_2$	$-3J_1-8J_2$
$-2J_1-7J_2$	$\frac{7}{2}J_1-9J_2$	$-5J_1-3J_2$	$-\frac{3}{2}J_1-7J_2$	$-3J_1-9J_2$
$-2J_1-8J_2$	$\frac{9}{2}J_1-2J_2$	$-5J_1-4J_2$	$-\frac{3}{2}J_1-8J_2$	$-3J_1-12J_2$
$-2J_1-9J_2$	$\frac{9}{2}J_1-3J_2$	$-5J_1-6J_2$	$-\frac{3}{2}J_1-9J_2$	$\frac{7}{2}J_1-7J_2$



$-2J_1-10J_2$	0	$-\frac{9}{2}J_1-5J_2$	$-\frac{5}{2}J_1-5J_2$	$-\frac{5}{2}J_1-2J_2$
$-3J_1-3J_2$	$-\frac{9}{2}J_1-6J_2$	$-2J_1$	$-\frac{5}{2}J_1-3J_2$	—
$-3J_1-4J_2$	$-\frac{9}{2}J_1-7J_2$	$-6J_1$	$-\frac{5}{2}J_1-8J_2$	—
$-3J_1-6J_2$	$-\frac{9}{2}J_1-9J_2$	$-3J_1-10J_2$	$-\frac{5}{2}J_1-7J_2$	—

Definition 1. The configuration φ is called the ground state for the Hamiltonian (1)

If $U(\varphi_b) = \min_{1 \leq i \leq 10} \{U_i\}$ for any $b \in M$.

References

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